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On the fermionic *Grande Bouffe*: more on higher spin symmetry breaking in AdS/CFT

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Abstract: We discuss fermionic higher spin gauge symmetry breaking in AdS space from a holographic perspective. Analogously to the recently discussed bosonic case, the higher spin Goldstino mode responsible for the symmetry breaking has a non-vanishing mass in the limit in which the gauge symmetry is restored. This result is precisely in agreement with the AdS/CFT correspondence, which implies that $\mathcal{N}=4$ SYM at vanishing coupling constant is dual to a theory in AdS which exhibits higher spin gauge symmetry enhancement. When the SYM coupling is non-zero, the current conservation condition becomes anomalous, and correspondingly the local higher spin symmetry in the bulk gets spontaneously broken. We also show that the mass of the Goldstino mode is exactly the one predicted by the correspondence. Finally, we obtain the form of a class of fermionic higher spin currents in the SYM side.

Keywords: Space-Time Symmetries, Supersymmetry and Duality, AdS-CFT and dS-CFT Correspondence.

Contents

1. Introduction

The AdS/CFT correspondence [1] is a conjectured duality relating a conformal field theory (CFT) in d dimensions to a field theory describing higher spin fields coupled to gravity in an AdS background in $D = d + 1$ dimensions. The most famous and remarkable example of this is the duality between IIB superstring theory on $AdS_5 \times S^5$ with N units of 5-form flux and SU(N) $\mathcal{N} = 4$ SYM theory in $d = 4$. The conjecture is most often discussed in the limit of large AdS radius where the higher spin fields become extremely massive and decouple from the (super)gravity modes. In this limit the AdS side is under control whereas the CFT side is not well understood since it corresponds to the limit of large 't Hooft coupling. Therefore one can make predictions for the strongly coupled CFT using the correspondence but these can generally only be checked for certain protected objects.

More recently the opposite limit, in which the CFT is weakly coupled, has been discussed by a number of people [2] and it is possible to extrapolate the string spectrum and precisely match it with the operator spectrum of free $\mathcal{N}= 4$ SYM in the planar limit [3]. In particular the limit of zero YM coupling has been conjectured to be dual to a massless higher spin field theory which, although inconsistent when coupled to gravity in flat space-time, can be consistently defined in AdS spaces [4] (for a review see [5] and references therein). When on the YM side the coupling is turned on, in AdS all the massless higher spin fields should develop a mass via the Stückelberg mechanism, essentially by eating lower spin Goldstone fields. This phenomenon was termed 'La Grande Bouffe' in [6]. The remaining massless fields will all be contained in the supergravity multiplet. On the other hand, in the dual CFT at zero coupling there will be infinitely many higher spin conserved currents (in one-to-one correspondence with the AdS higher spin gauge fields). The CFT counter-part of 'La Grande Bouffe' is the anomalous violation of these conserved currents when the coupling is turned on, with the only remaining conserved currents lying in the energy momentum multiplet. The most famous example of this anomalous violation of classically conserved currents is given by the Konishi multiplet.

The simplest example of a (bosonic) higher spin s field is that of a tensor with s completely symmetrised spacetime indices. For such an object, in flat spacetime, the massless limit of a massive spin s field gives rise to $s+1$ massless fields of spins $0, 1, \ldots s$. However the AdS/CFT correspondence predicts that the massless limit of a massive spin s field in AdS is a massless spin s field and a massive spin $s-1$ field. The reason for this is that HS currents $J_{i_1 \ldots i_s}$ with $s > 2$ occur in $\mathcal{N} = 4$ SYM, where they are conserved at vanishing coupling $g = 0$, and conformal invariance fixes the dimension of such a spin s conserved current on the d dimensional boundary to be $s + d - 2$. Interactions are responsible for their anomalous violation

$$\partial^{i_1} J_{i_1 \ldots i_s} = g \mathcal{X}_{i_2 \ldots i_s} ,$$

and in the zero coupling limit the dimension of $\mathcal{X}$ is $s + d - 1$. This implies that $\mathcal{X}$ is not a conserved spin $s - 1$ current when $g = 0$, and therefore one expects it to be dual to a massive field in the bulk.

In a recent paper [7] the Stückelberg formulation of bosonic massive higher spin fields (with completely symmetrised spacetime indices) in AdS was derived. The method consisted in obtaining the field equations in flat space via dimensional reduction of the massless equations in one dimension higher, and then including the necessary counterterms in order to find the equivalent equations in AdS (see also [8, 9] for similar results using a different method). One can then use these equations in AdS to extrapolate the massless limit, and one indeed obtains a massless spin s field and a massive spin $s - 1$ field in line with the CFT predictions (this phenomenon has also been discussed from a cosmological viewpoint in [8] where it was termed 'partial masslessness'). The mass of the spin $s - 1$ field one obtains in this way is precisely the one predicted by AdS/CFT.

In this note we wish to continue this study by considering fermionic higher spin currents. We again try to covariantise the Stückelberg formulation of massive fermionic higher spin fields from flat space [10] to AdS space. We will only focus on higher spin fermions whose spacetime indices are completely symmetrised. The fermionic case turns out to be quite different from the bosonic case, both in the auxiliary field structure and in the fact that here the gauge transformations themselves must be modified on covariantising from flat-space to AdS.

This note is organised as follows. In section 2 we consider the Stückelberg formulation of massive higher spin fermion field equations in D-dimensional flat spacetime. The equations are formally derived from $D+1$ dimensions, giving an exponential dependence on the $D + 1$-th coordinate to the field [10]. In section 3 we consider the same equations in AdS, and we show that gauge invariance requires the inclusion of additional counterterms to the equations, such that one gets a massless spin s field and a *massive* spin $s - 1$ field from a massive spin s field in the limit of zero mass. We also show that, exactly as in the bosonic case, the mass of the spin $s-1$ field is precisely in agreement with the one predicted by the AdS/CFT correspondence. Section 4 contains a discussion and the conclusions, focusing in particular on the case in which $D = 5$ (and $d = 4$), corresponding to the duality relating superstring theory on $AdS_5 \times S^5$ to $\mathcal{N} = 4$ SYM. All currents in the free theory are

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classified and examples of the fermionic currents dual to the fermionic higher spin fields are given together with their anomalous equations.

2. Higher-spin fermions in flat spacetime

In this section we study the Stückelberg formulation of massive higher spin fermion field equations in flat spacetime. In analogy with the bosonic case, analysed in [7], we obtain the massive equations in D dimensions via dimensional reduction of the massless equations in $D+1$ dimensions. We restrict our attention to spinors that are completely symmetric with respect to the spacetime indices. Reality properties of the spinors are not relevant for this analysis. Moreover, in the interesting case of odd D, we assume that the $D+1$ dimensional spinor is Weyl,[1] so that the D dimensional one is Dirac. If instead D is even, both the $D+1$ and the D dimensional spinors are Dirac. The massless equations are

$$\gamma \cdot \partial \Psi_{M_1 \dots M_n} - n\, \partial_{(M_1} (\gamma \cdot \Psi)_{M_2 \dots M_n)} = 0\,, \tag{2.1}$$

where the tensor-spinor Ψ is completely symmetric with respect to the spacetime indices, and satisfies the condition

$$(\gamma \cdot \Psi)^M{}_M = 0\,. \tag{2.2}$$

The equations are invariant with respect to the gauge transformation

$$\delta \Psi_{M_1 \dots M_n} = n \partial_{(M_1} \epsilon_{M_2 \dots M_n)}\,, \tag{2.3}$$

where ϵ is a spinor symmetric in its spacetime indices, and satisfies the constraint

$$\gamma \cdot \epsilon = 0\,. \tag{2.4}$$

These equations can be obtained from the gauge-invariant lagrangian [11]

$$\begin{aligned} \mathcal{L} = {} & \frac{1}{2} \bar{\Psi}^{(n)}_{\dots} \gamma^M \partial_M \Psi^{(n)}_{\dots} + \frac{n}{2} \bar{\Psi}^{(n)}_{M\dots} \gamma^M \gamma^N \partial_N \gamma^P \Psi^{(n)}_{P\dots} - \\ & - \frac{n(n-1)}{8} \bar{\Psi}^{(n)L}{}_{L\dots} \gamma^M \partial_M \Psi^{(n)N}{}_{N\dots} + \\ & + \frac{n(n-1)}{2} \bar{\Psi}^{(n)L}{}_{L\dots} \partial_M \gamma_N \Psi^{(n)MN}{}_{\dots} - n \bar{\Psi}^{(n)}_{M\dots} \gamma^M \partial_N \Psi^{(n)N}{}_{\dots}\,. \end{aligned} \tag{2.5}$$

In order to obtain the equations for a massive field in D dimensions in the Stückelberg formulation, we consider the field Ψ to depend harmonically on the $D+1$-th coordinate,

$$\Psi_{\mu_1 \dots \mu_{n-k} y \dots y}(x,y) = (i)^k e^{-i\frac{\pi}{4}\gamma_y} \psi^{(n-k)}_{\mu_1 \dots \mu_k}(x) e^{imy} + \text{c. c.}\,, \tag{2.6}$$

[1] In this case all $D+1$-dimensional γ-matrices in the equations below should be thought of as $D+1$ dimensional σ-matrices.

where γ_y is the gamma-matrix in the $D+1$-th direction. From eq. (2.1), we therefore obtain the equations

$$[\gamma^\mu \partial_\mu + m(1-k)]\psi^{(n-k)}_{\mu_1\dots\mu_{n-k}} - (n-k)\partial_{(\mu_1}(\gamma\cdot\psi^{(n-k)})_{\mu_2\dots\mu_{n-k})} - \\ -km(\gamma\cdot\psi^{(n-k+1)})_{\mu_1\dots\mu_{n-k}} - (n-k)\partial_{(\mu_1}\psi^{(n-k-1)}_{\mu_2\dots\mu_{n-k})} = 0\,, \tag{2.7}$$

where $k = 0, 1, \dots, n$, and from the gauge transformations of eq. (2.3) one gets

$$\delta\psi^{n-k}_{\mu_1\dots\mu_{n-k}} = (n-k)\partial_{(\mu_1}\epsilon^{(n-k-1)}_{\mu_2\dots\mu_{n-k})} + km\epsilon^{(n-k)}_{\mu_1\dots\mu_{n-k}}\,, \tag{2.8}$$

where the D-dimensional gauge parameters $\epsilon^{(n-k)}$ are defined by means of

$$\epsilon_{\mu_1\dots\mu_{n-k-1}y\dots y}(x,y) = (i)^k e^{-i\frac{\pi}{4}\gamma_y}\epsilon^{(n-k-1)}_{\mu_1\dots\mu_{n-k-1}}(x)e^{imy} + \text{c.c.}\,. \tag{2.9}$$

In D dimensions, the gamma-traceless condition of eq. (2.4) on ϵ becomes

$$\gamma^\mu\epsilon^{(n-k)}_\mu + \epsilon^{(n-k-1)} = 0\,, \qquad k = 1, \dots, n-1\,, \tag{2.10}$$

while the condition (2.2) on Ψ becomes

$$(\gamma\cdot\psi^{(n-k)})^\mu{}_\mu + \psi^{\mu(n-k-1)}_{\mu} - (\gamma\cdot\psi)^{(n-k-2)} - \psi^{(n-k-3)} = 0\,, \qquad k = 0, \dots, n-3\,. \tag{2.11}$$

If $m \neq 0$, one can use eq. (2.8) to gauge away some of the lower rank spinors, while the lower rank spinors that can not be gauged away because of the constraint (2.4) turn out to be identically zero on shell, and are the auxiliary fields of the massive theory. therefore, one ends up with an equation for a massive spin $s = n + 1/2$ field. In [12] a lagrangian formulation for spinors completely symmetric in their spacetime indices was given in terms of a field $\psi^{(n)}$ satisfying the gamma-traceless condition $\gamma\cdot\psi = 0$. The condition that the divergence of this field vanishes on shell is realised by means of a set of auxiliary fields $\psi^{(n-1)}$, $\psi^{(n-k)}$, $\chi^{(n-k)}$, with $k = 2, \dots, n$, all satisfying a gamma-traceless condition. In [10] it was shown that these auxiliary fields are precisely the fields that can not be gauged away in the Stückelberg formulation. Let us consider for simplicity the case of spin 5/2, i.e. $n = 2$. One can use the gauge parameter $\epsilon^{(1)}$ to gauge away $\psi^{(1)}$ completely, while $\epsilon^{(0)}$ can not be used because it is related to $\epsilon^{(1)}$ via eq. (2.4). We are therefore left with $\psi^{(2)}$ and $\psi^{(0)}$, and extracting the gamma-traceless part from $\psi^{(2)}$ one gets exactly the auxiliary field structure of [12]. It is straightforward to see that a similar analysis works for any n.

If $m = 0$, the situation is different, since none of the gauge parameters can be used to gauge away any of the fields, and therefore all the fields $\psi^{(n-k)}$, $k = 0, 1, \dots, n$ become massless. In order to see this from our equations, one has to perform recursive field redefinitions, so that eq. (2.10) becomes a gamma-traceless condition for the redefined parameters, while eq. (2.11) becomes a condition of the form (2.2) for the redefined fields, whose gauge transformations look exactly like eqs. (2.8) with $m = 0$ in terms of the new parameters.

As we will see in the next section, and analogously to the bosonic case, in AdS the massless limit of the Stückelberg equations gives a completely different result, since imposing masslessness for $\psi^{(n)}$ will result in a mass term for $\psi^{(n-1)}$.

3. Higher-spin fermions in AdS

In this section we want to consider the AdS equivalent of the flat space Stückelberg equations of the previous section for $m \neq 0$, and then study the limit of vanishing m. In analogy with the bosonic case [7] and for consistency with holography, we expect that the gauge fixing procedure that leads to an equation for a massive spin s field when $m \neq 0$ can still take place up to spin $s-1$ when $m = 0$, so that the resulting equations in this limit describe a massless spin $s = n + 1/2$ fermionic field $\psi^{(n)}$ and a *massive* spin $s - 1 = n - 1/2$ fermionic field $\chi^{(n-1)}$.

In analogy with the bosonic case [7], we consider eq. (2.7) for $k = 1$, and we gauge away $\psi^{(n-2)}$ using $\epsilon^{(n-2)}$. Therefore, only $\psi^{(n)}$ and $\psi^{(n-1)}$ (that will be denoted with $\chi^{(n-1)}$ from now on) will appear in the equation, and the only gauge invariance left is the one with respect to the *gamma-traceless* parameter $\epsilon^{(n-1)}$. We will see that this gauge invariance in AdS will require the addition of a mass term for $\chi^{(n-1)}$. We will then consider the limit of vanishing m, and we will see that this mass term differs from the AdS mass term[2] for $\chi^{(n-1)}$. This result implies that the field $\chi^{(n-1)}$ is massive in the limit in which $\psi^{(n)}$ becomes a massless field.

We now derive first the AdS mass for a generic spin $s = l + 1/2$ field. We therefore consider the equation

$$\gamma \cdot \nabla \psi_{\mu_1 \ldots \mu_l} - l \, \nabla_{(\mu_1} (\gamma \cdot \psi)_{\mu_2 \ldots \mu_l)} + M_{AdS} \psi_{\mu_1 \ldots \mu_l} + \tilde{M}_{AdS} \gamma_{(\mu_1} (\gamma \cdot \psi)_{\mu_2 \ldots \mu_l)} = 0 \,, \tag{3.1}$$

where $\psi^{(l)}$ satisfies the constraint $(\gamma \cdot \psi^{(l)})^{\mu}{}_{\mu} = 0$, and its gauge transformation is

$$\delta \psi^{(l)}_{\mu_1 \ldots \mu_l} = l \nabla_{(\mu_1} \epsilon^{(l-1)}_{\mu_2 \ldots \mu_l)} + M' \gamma_{(\mu_1} \epsilon^{(l-1)}_{\mu_2 \ldots \mu_l)} \,, \tag{3.2}$$

where ϵ is gamma-traceless, and we want to determine M_{AdS}, $\tilde{M}_{AdS}$ and M' such that eq. (3.1) is gauge invariant with respect to the transformation of eq. (3.2). Using the fact that the commutator of two covariant derivatives acting on a spinor with $l - 1$ vector indices is

$$[\nabla_\mu, \nabla_\nu] \epsilon_{\rho_1 \ldots \rho_{l-1}} = -\frac{1}{2L^2} \gamma_{\mu\nu} \epsilon_{\mu_1 \ldots \mu_{l-1}} + \frac{l-1}{L^2} (g_{\nu(\rho_1} \epsilon_{\rho_2 \ldots \rho_{l-1})\mu} - g_{\mu(\rho_1} \epsilon_{\rho_2 \ldots \rho_{l-1})\nu}) \,, \tag{3.3}$$

where L is the AdS radius, one obtains

$$M' = \frac{l}{2L} \,, \tag{3.4}$$

$$M_{AdS} = \frac{1}{2L} [D + 2(l-2)] \,, \tag{3.5}$$

$$\tilde{M}_{AdS} = \frac{l}{2L} \,. \tag{3.6}$$

Since the gamma-trace of the field can always be put to zero choosing a suitable gauge, the relevant AdS mass term is M_{AdS} in eq. (3.5).

[2] Recall that in AdS one defines a field to be massless if its equation of motion is gauge invariant. As we will see, such an equation contains a mass-like term which we call the AdS mass.

We now consider the equation for the spin $n+1/2$ field $\psi^{(n)}$ and the spin $n-1/2$ Stückelberg field $\chi^{(n-1)}$, after having gauged away $\psi^{(n-2)}$ by means of $\epsilon^{(n-2)}$. The covariantisation of eq. (2.7) for k=0 takes the form of eq. (3.1) together with mass terms for $\psi^{(n)}$ and a term involving the derivative of the spin $n-1/2$ Stückelberg field $\chi^{(n-1)}$,

$$\begin{aligned} \gamma\cdot\nabla\psi^{(n)}_{\mu_1\ldots\mu_n} - n\ \nabla_{(\mu_1}(\gamma\cdot\psi^{(n)})_{\mu_2\ldots\mu_n)} + M_{AdS}\psi^{(n)}_{\mu_1\ldots\mu_n} + \\ +\tilde{M}_{AdS}\gamma_{(\mu_1}(\gamma\cdot\psi^{(n)})_{\mu_2\ldots\mu_n)} + m\,\psi^{(n)}_{\mu_1\ldots\mu_n} - n\nabla_{(\mu_1}\chi^{(n-1)}_{\mu_2\ldots\mu_n)} - \frac{n}{2L}\gamma_{(\mu_1}\chi^{(n-1)}_{\mu_2\ldots\mu_n)} = 0\,. \end{aligned} \quad (3.7)$$

This equation is gauge invariant under

$$\delta\psi^{(n)}_{\mu_1\ldots\mu_n} = n\nabla_{(\mu_1}\epsilon_{\mu_2\mu_n)} + \frac{n}{2L}\gamma_{(\mu_1}\epsilon_{\mu_2\ldots\mu_n)}\,, \qquad \delta\chi^{(n-1)}_{\mu_1\ldots\mu_{n-1}} = m\epsilon_{\mu_1\ldots\mu_{n-1}}\,. \quad (3.8)$$

Indeed the first and second lines are separately gauge invariant: the first line is simply the equation for a massless spin $n+1/2$ field in AdS derived above, whereas in the second line there is a mass term for $\psi^{(n)}$ whose gauge variation is cancelled by the gauge variation of the terms involving $\chi^{(n-1)}$.

In order to derive the equation for $\chi^{(n)}$, we consider the expression that we get from eq. (2.7) for $k=1$,

$$\gamma^\mu\nabla_\mu\chi^{(n-1)}_{\mu_1\ldots\mu_{n-1}} - (n-1)\nabla_{(\mu_1}(\gamma\cdot\chi^{(n-1)})_{\mu_2\ldots\mu_{n-1})} - m(\gamma\cdot\psi^{(n)})_{\mu_1\ldots\mu_{n-1}}\,. \quad (3.9)$$

Its variation with respect to the gauge transformation (3.8) with ϵ gamma-traceless, is

$$-\frac{m}{2L}[D+2(n-1)]\epsilon_{\mu_1\ldots\epsilon_{n-1}}\,, \quad (3.10)$$

and in order to cancel this, one needs to include the mass term

$$M\chi^{(n-1)} = \frac{1}{2L}[D+2(n-1)]\chi^{(n-1)}\,, \quad (3.11)$$

so that the resulting equation is

$$\begin{aligned} \gamma^\mu\nabla_\mu\chi^{(n-1)}_{\mu_1\ldots\mu_{n-1}} - (n-1)\nabla_{(\mu_1}(\gamma\cdot\chi^{(n-1)})_{\mu_2\ldots\mu_{n-1})} - \\ -m(\gamma\cdot\psi^{(n)})_{\mu_1\ldots\mu_{n-1}} + \frac{1}{2L}[D+2(n-1)]\chi^{(n-1)}_{\mu_1\ldots\mu_{n-1}} = 0\,. \end{aligned} \quad (3.12)$$

If $m\neq 0$, one can still gauge away the Stückelberg field $\chi^{(n-1)}$, so that only a massive spin s field remains. If $m=0$, though, it is clear from eq. (3.8) that this is no longer possible. What we are going to show now is that one can consistently consider the $m\to 0$ limit of Equations (3.7), (3.12), since all the lower rank fields can still be gauged away in this limit.

Indeed, the fact that $M\neq M_{AdS}$ proves that in the limit of vanishing m, in which the field $\psi^{(n)}$ becomes massless, the field $\chi^{(n-1)}$ remains a massive field. The difference between the square of the mass M and the square of the AdS mass M_{AdS} for $\chi^{(n-1)}$, obtained from (3.5) with $l=n-1$, is

$$M^2L^2 - M^2_{AdS}L^2 = 2D+4n-8\,. \quad (3.13)$$

This result is in perfect agreement with the AdS/CFT relation [13]

$$M^2L^2 - M^2_{AdS}L^2 = \Delta(\Delta - d) - \Delta_{\rm min}(\Delta_{\rm min} - d)\,, \tag{3.14}$$

where $d = D - 1$ and Δ is the dimension of the operator dual to $\chi^{(n-1)}$, whose value is

$$\Delta = d + n - \frac{1}{2} \tag{3.15}$$

in the limit of vanishing Yang-Mills coupling, while $\Delta_{\rm min} = d + n - \frac{5}{2}$ represents the conformal unitary bound for the dimension of a spin $n - 1/2$ operator. Finally, as in the flat spacetime case, a field redefinition for $\psi^{(n)}$ is needed in order to recover the standard massless equation for a spin $s = n + 1/2$ field in AdS when $m = 0$.

4. Discussion and conclusions

Free $\mathcal{N} = 4$ SYM contains many spin s conserved currents which conformal representation theory tells us must have dimension $\Delta = s + 2$. The fundamental fields of the theory, $\phi_I, \lambda^A_\alpha, \bar\lambda_{\dot\alpha A}, F_{ij}$ all have twist $(\Delta - s)$ one, and since any composite operator must have twist greater than or equal to the sum of the twists of its constituent fundamental fields in the free theory, it follows that all currents must be bilinear in the fundamental fields and are thus elements of the higher spin doubleton multiplet [14]. Indeed it turns out that all bilinear conformal primary operators have twist two and so the following statement is true: all currents belong to the higher spin doubleton multiplet and conversely all primary operators in the doubleton multiplet with spin $j_1 > 0$, $j_2 > 0$ are currents.

The higher spin doubleton multiplet splits into infinitely many representations of the superconformal group, namely the energy-momentum supermultiplet $\mathcal{T}_{IJ} := {\rm Tr}(W_IW_J - 1/6\delta_{IJ}W_KW_K)$, the Konishi supermultiplet $\mathcal{H} := {\rm Tr}(W_IW_I)$ and the higher spin analogues of the Konishi supermultiplet [15] $\mathcal{H}^{(s)} \sim {\rm Tr}(W_I\partial^sW_I) + \cdots$, $s \in 2\mathbb{Z}$ (the precise form of the latter are given in [7] where one can also find details about our notation). Since all component currents in $N = 4$ SYM are contained in these multiplets they are all of the form

$$D_{(\alpha_1A_1}\dots D_{\alpha_kA_k}\bar D^{A_1}_{(\dot\alpha_1}\dots\bar D^{A_l}_{\dot\alpha_l}\,\mathcal{H}^{(s)}_{\alpha_{k+1}\dots\alpha_{k+s})\dot\alpha_{l+1}\dots\dot\alpha_{l+s}}|_{\theta=\bar\theta=0} \qquad 0 \le k, l \le 4 \tag{4.1}$$

or are currents contained in the energy-momentum multiplet. One can thus straightforwardly classify all currents in the theory (see table 1 below and also [16]).

In the interacting $N = 4$ SYM theory, all higher spin currents, other than those in the energy-momentum supermultiplet, become anomalous meaning their current conservation condition is violated $\partial^{i_1}J_{i_1\dots i_s} = g\mathcal{X}_{i_2\dots i_s}$. The anomalous (non)-conservation equations for these currents are encoded in analogous anomalous equations for the $N = 4$ superfields $\mathcal{H}_{\alpha_1\dots\alpha_s\dot\alpha_1\dots\dot\alpha_s}$ (supercurrents) containing these currents. The relevant superfield equations are

$$D^\alpha_A\mathcal{H}^{(s)}_{\alpha\dots} = g\mathcal{M}_{A\dots} \qquad \bar D^{\dot\alpha A}\mathcal{H}^{(s)}_{\dot\alpha\dots} = g\bar{\mathcal{M}}^A_{\dots} \qquad s \ge 2 \tag{4.2}$$

$$D_{\alpha A}D^\alpha_B\mathcal{H}^{(0)} = g\mathcal{M}_{AB} \qquad \bar D^A_{\dot\alpha}D^{\dot\alpha B}\mathcal{H}^{(0)} = g\bar{\mathcal{M}}^{AB}\,. \tag{4.3}$$

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Current	Spin(j_1, j_2)	$\mathcal{H}^{(s)} : s =$	(k,l)	SU(4)rep.
$J_{(i_1 \ldots i_n)}$	$(\frac{n}{2}, \frac{n}{2})$	n	$(0,0)$	$\mathbf{1}$
		$n-1$	$(1,1)$	$\mathbf{4} \times \bar{\mathbf{4}} = \mathbf{15} + \mathbf{1}$
		$n-2$	$(2,2)$	$\mathbf{6} \times \mathbf{6} = \mathbf{20}' + \mathbf{15} + \mathbf{1}$
		$n-3$	$(3,3)$	$\bar{\mathbf{4}} \times \mathbf{4} = \mathbf{15} + \mathbf{1}$
		$n-4$	$(4,4)$	$\mathbf{1}$
$\Psi_{\dot{\alpha}(i_1 \ldots i_n)}$	$(\frac{n}{2}, \frac{n+1}{2})$	n	$(0,1)$	$\bar{\mathbf{4}}$
		$n-1$	$(1,2)$	$\mathbf{4} \times \mathbf{6} = \mathbf{20} + \bar{\mathbf{4}}$
		$n-2$	$(2,3)$	$\mathbf{6} \times \mathbf{4} = \mathbf{20} + \bar{\mathbf{4}}$
		$n-3$	$(3,4)$	$\bar{\mathbf{4}}$
$J_{[kj](i_1 \ldots i_n)}$	$(\frac{n}{2}, \frac{n}{2}+1)$	n	$(0,2)$	$\mathbf{6}$
		$n-1$	$(1,3)$	$\mathbf{4} \times \mathbf{4} = \mathbf{10} + \mathbf{6}$
		$n-2$	$(2,4)$	$\mathbf{6}$
$\Psi_{\dot{\alpha}[jk](i_1 \ldots i_n)}$	$(\frac{n}{2}, \frac{n+3}{2})$	n	$(0,3)$	$\mathbf{4}$
		$n-1$	$(1,4)$	$\mathbf{4}$
$J_{[jk][lm](i_1 \ldots i_n)}$	$(\frac{n}{2}, \frac{n}{2}+2)$	n	$(0,4)$	$\mathbf{1}$

Table 1: Table giving all currents (up to complex conjugation) in $N = 4$ SYM (apart from those in the energy-momentum multiplet), the supermultiplets they appear in and the SU(4) representations they carry. The numbers (k, l) indicate where the currents lie in the supermultiplet according to ¡(4.1). These operators are only currents for $j_1 > 0$, $j_2 > 0$. We must also have $s \geq 0$ and even. The mixed symmetry tensors lie in irreducible representations of the Lorentz group given by Young tableaux (4.6)–(4.8). Finally, the complete current content of $N = 4$ SYM must include the energy-momentum multiplet which contains 1 real spin $(1,1)$ current, $\mathbf{4}$ complex spin $(1, 1/2)$ currents, and $\mathbf{15}$ spin $(1/2, 1/2)$ currents.

Here $\mathcal{H}$, $\mathcal{M}, \bar{\mathcal{M}}$ are separate $N = 4$ supermultiplets in the free theory ($g = 0$) whereas in the interacting theory we can see that they combine and indeed they further combine with another supermultiplet $\mathcal{N}$ to form one long supermultiplet.[3]

In [7] the Stückelberg AdS equations for massive bosonic spin s fields, completely symmetric in their spacetime indices, was given. These fields are the holographic duals of the currents $J_{(i_1 \ldots i_n)}$ in the first row of table 1. In the limit of zero mass for the spin s field, a massive spin $s - 1$ field appears, whose mass is exactly in agreement with the AdS/CFT correspondence. In this note we have studied the same mechanism for the fields dual to the fermionic currents $\Psi_{\dot{\alpha}(i_1 \ldots i_n)}$ in the second row of table 1. The simplest example of such fermionic currents is the first supersymmetric descendant of the higher spin analogues of the Konishi supermultiplet,

$$\lambda^{(n)}_{A\alpha_0 \ldots \alpha_n \dot{\alpha}_1 \ldots \dot{\alpha}_n} := D_{(\alpha_0 A} \mathcal{H}^{(n)}_{\alpha_1 \ldots \alpha_n)\dot{\alpha}_1 \ldots \dot{\alpha}_n} \,. \tag{4.4}$$

This is the descendant with $(k, l) = (1, 0)$ (the complex conjugate of the spin $(n/2, n/2 + 1/2)$ current in the $\bar{\mathbf{4}}$ representation in table 1, for which $s = n$). Here n is even and non-vanishing and is the spin of the top component of the multiplet while the spin of $\lambda^{(n)}$

[3] The precise forms of $\mathcal{M}, \bar{\mathcal{M}}$ and $\mathcal{N}$ are given in [7, 17].

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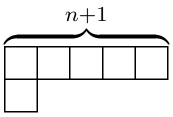
is $n+1/2$. The violation of the current conservation equation can be found using standard superfield methods from the above superfield equations (4.2) to be

$$\begin{aligned}4\partial^{\alpha_0\dot\alpha_0}\lambda_{B\alpha_0\ldots\alpha_s\dot\alpha_0\ldots\dot\alpha_{s-1}} = {} & \frac{g}{2}\partial_{(\alpha_1}{}^{\dot\alpha_0}\mathcal{M}_{B\alpha_2\ldots\alpha_s)\dot\alpha_0\ldots\dot\alpha_{s-1}}|_{\theta=\bar\theta=0} - \\ & -igD_{(\alpha_0 B}D^{\alpha_0}_{A}\bar{\mathcal{M}}^{A}_{\alpha_1\ldots\alpha_s\dot\alpha_1\ldots\dot\alpha_{s-1}}|_{\theta=\bar\theta=0} + \\ & +\frac{ig}{2}[\bar D^{\dot\alpha_0 A}, D_{(\alpha_1 B}]\mathcal{M}_{A\alpha_2\ldots\alpha_s)\dot\alpha_0\ldots\dot\alpha_{s-1}}|_{\theta=\bar\theta=0}\,. \end{aligned} \tag{4.5}$$

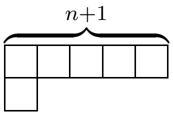
As in the bosonic case, we have shown that a massive fermion field of spin s in AdS becomes a massless spin s field and a massive spin $s-1$ field. We computed the mass of the spin $s-1$ field, showing that it exactly agrees with the dimension of the dual operator in SYM in the limit of vanishing coupling.

The above fermionic currents have spin $2k+1/2$ for k a positive integer. For vanishing k, corresponding to the Konishi multiplet, one obtains a spin 1/2 operator which can never be a conserved current. There are also fermionic currents of spin $2k+3/2$ the simplest case being the spin 3/2 case in the Konishi multiplet. Indeed there are two spin 3/2 currents in $\mathcal{N}$=4 SYM, the supersymmetric current in the energy momentum tensor multiplet which is also conserved in the interacting theory, and the descendant of the Konishi which is only conserved in the free theory.

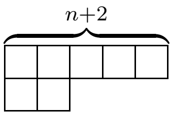
It would be interesting to study the same mechanism for the fields dual to the remaining currents in table 1. The Young tableaux associated to them are



, (4.6)



$_{\alpha}$, (4.7)



, (4.8)

corresponding respectively to the last three rows of the table. The field strengths of these fields satisfy self-duality conditions that are generalisations of the self-duality condition of the antisymmetric 2-forms in the supergravity multiplet [14]. Massive fields of mixed symmetry in AdS_5 were discussed in [18], while flat space equations for gauge fields in arbitrary representations of the Lorentz group were introduced in [19] and more recently discussed in [20]. We hope to soon report on progress in this direction.

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